COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Extended self-similarity in turbulent flows"

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A recent paper [R. Benzi, S. Ciliberto, R. Tripiccione, C. Baudet, F. Massaioli, and S. Succi, Phys. Rev. E **48**, R29 (1993)] concluded that the scaling of turbulent structure functions extends deeper into the viscous subrange than what has been assumed so far. We comment on this notion of "extended self-similarity." We verify the (approximate) universal character of the viscous subrange scaling of structure functions but show that it is different from their inertial range scaling. The effect is small (although significant) in laboratory flows and it has profound consequences for the multifractal model.

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The structure function $G_p(r)$ quantifies the statistics of the small-scale structure of turbulence. It is defined as $G_p(r) = \langle \Delta u^p \rangle_x$, where $\Delta u(r) = u(x+r) - u(x)$ is a velocity difference over a distance r; u and r point in the same direction. The structure function has scaling behavior, $G_p(r) \simeq r^{\zeta(p)}$, over a range of distances $l < r < L$ that is bounded from below by the action of viscosity, $l \approx 30\eta$, and from above by an external length scale L. The interval $[l, L]$ is called the inertial range because viscosity is not yet felt and the transport of energy is through inertial terms in the Navier-Stokes equation. The energy dissipation ϵ and the viscosity ν define the Kolmogorov scale $\eta = (\nu^3\ /\epsilon)^{1/4}$. The well-known p dependence of the scaling exponent $\zeta(p) = p/3$ is exact for $p = 3$ in the limit of zero viscosity [1]. For large values of p, deviations of Kolmogorov's prediction, $\zeta(p) = p/3$, have been found; these deviations have prompted the multifractal model by Parisi and Frisch [2].

The problem in laboratory experiments at moderate Reynolds numbers ($R_\lambda \approx 10^2$, based on a correlation scale λ) is that the inertial range is small and it is often hard to recognize scaling behavior at all. In a recent paper Benzi *et al.* [4] find a much improved scaling by plotting $G_p(r)$ as a function of $\langle |\Delta u|^3 \rangle$. The resulting scaling range seemingly extends to much smaller values of r, $r \approx \eta$, a circumstance that prompted the notion of "extended self-similarity." A second observation was that the behavior of $G_p(r)$ for $r \lesssim 30\eta$ does not depend on R_λ, and, therefore, would be *universal.*

In this Comment we will show that their second observation is compatible with experiments in a modest range of R_λ. We will also argue that, although "extended self-similarity" may appear to be a good approximation to the behavior of structure functions in the inertial range, there are significant deviations. At small values of r, $r \lesssim 30\eta$, the structure functions display a behavior that is *different* from their $30\eta \lesssim r < L$ inertial range scaling. Specifically, the structure function can be written as $G_p(r) = [r f_p(r)]^{\zeta(p)}$, with a function $f_p(r)$ that, in the viscous subrange, $r \lesssim 30\eta$; *essentially depends on p.* "Extended similarity," instead, is equivalent to the statement that $f_p(r)$ is *independent* of p. The function f_p is the deviation of the structure function from ideal scaling, $G_p(r) \sim r^{\zeta(p)}$. It is a convex function that bends down into the viscous subrange. It is a trivial observation that a plot of $\log G_p(r)$ as a function of, for example, $\log G_3(r)$ is only a straight line for all p if $f_p(r)$ is the same for all p. The p dependence of the residual functions $f_p(r)$ that we find is in agreement with multiscaling, an inherent property of multifractals [3]. The key point is that stronger singularities of the turbulent velocity field [that are expressed in the $G_p(r)$ at larger p] extend deeper into the viscous scales as p increases. Therefore the scaling behavior of structure functions of increasing order extends to smaller r as p increases.

At modest values of R_λ (5×10^2–10^3) a scaling region can often be recognized in a log-log plot of $G_3(r)$, however, it is not well defined. The standard procedure [5] then is to select the interval bounds such that the straight line fitted to $\log G_3(\log r)$ in this interval has slope $\zeta(3) = 1$. This normalizes the structure functions $G_p(r)$. It often amounts to a sacrifice of the quality of the fit to the requirement $\zeta(3) = 1$.

Several other, more explicit ways of normalizing structure functions have in the past been attempted to improve their scaling behavior. One way is to divide structure functions of order p by a low-order moment k, for example, $k = 3$,

$$H_p(r) = G_p(r)/G_k(r). \quad (1)$$

The normalization discussed by Benzi *et al.* [4] is to plot $G_p(r)$ as a function of $G_k(r)$, using r as parameter,

$$I_p(r) = G_p(\alpha),\ \alpha = G_k(r). \qquad (2)$$

In all three instances, the value of the measured exponents $\zeta(p)$ is always relative to the assumed scaling behavior of $G_3(r)$. There are many experimental reasons for (small) deviations of $\zeta(3)$ from unity, for example, deviations from isotropy, homogeneity, or equilibrium of the flow. It is a question, however, whether these defects may be cured by normalization.

The normalized structure function $H_p(r)$ would be most appropriate if the structure functions are of the form $G_p(r) = f(r)r^{\zeta(p)}$ whereas the normalization I_p is more appropriate if $G_p(r) = [f(r)r]^{\zeta(p)}$. The latter normalization is one for the *derivative* of the structure function. It dates back to the problem of laser beam propagation through turbulent media and was first tried in the context of turbulent structure functions in Ref. [6]. By using this normalization, Benzi *et al.* [4] come to the conclusion that $G_p(r)$ is of the form $G_p(r) = [f(r)r]^{\zeta(p)}$. While this form is indeed strongly suggested by the data, we will demonstrate that the function $f(r)$ essentially depends on p.

The problem is that of all normalizations considered here, the implicit normalization $I_p(r)$ is least sensitive to the shape of $f_p(r)$ (and, therefore, produces the best overall scaling). Let us define the residue of the structure function as that which remains if we subtract the constant $\zeta(p)$ from the local slope of G_p, H_p, or I_p in a log-log plot. For the three normalizations considered here, these residues are, respectively,

$$R^{(G)}(\ln r) = \zeta(p) f_p', \qquad (3a)$$

$$R^{(H)}(\ln r) = \zeta(p) f_p' - \zeta(k) f_k', \qquad (3b)$$

$$R^{(I)}(\ln r) = \frac{\zeta(p)}{\zeta(k)} \frac{f_p' - f_k'}{1 + f_k'}, \qquad (3c)$$

where $f_p' = d \ln f_p / d \ln r$. The function $f_p(r)$ is convex, therefore $R^{(I)}$ is the smallest of the three residues. The conclusion is that from a plot involving normalization I_p one would be tempted to conclude a function $f_p(r)$ that is the same for all p. Only in the case of very good statistical accuracy would one recover the p dependence of $f_p(r)$.

The statistical accuracy of high order structure functions is notoriously problematic. For increasing order p, $G_p(r)$ is an average over the increasingly rare instances of increasingly large velocity differences. We have built a special digital device that can measure high-order structure functions in real time through accumulated probability distribution functions of $\Delta u(r)$. The maximum number of velocity samples that we have taken is 1.5×10^9, two orders of magnitude larger than what has been customary so far. Not only has this significantly eased the statistics problem, but the absence of stored time series and the immediate availability of the structure function has allowed systematic experimentation with the flow conditions.

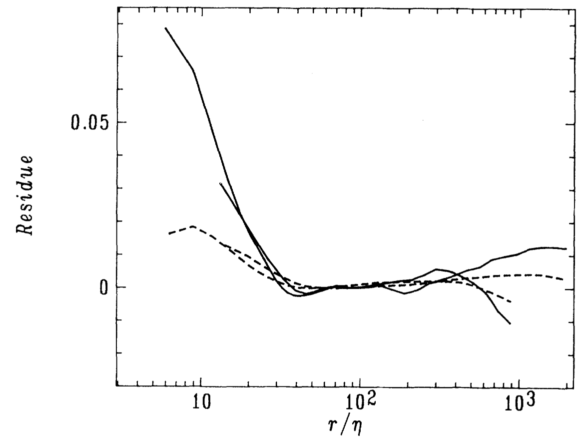

FIG. 1. The residual function $\log_{10}(f_p/f_2)$ for $p = 4$ (dashed lines) and $p = 8$ (full lines) for a turbulent jet flow and for grid-generated turbulence. The mean air velocities were $U = 12$ m/s and 11 m/s and the Kolmogorov lengths η were 9.5×10^{-5} m and 1.9×10^{-4} m, respectively. Therefore the lines for the grid turbulence are the ones that extend deepest into the viscous subrange. The fact that for $r < 30\eta$ the higher moment is larger indicates that f_p becomes flatter.

Figure 1 shows the residue, $\log_{10}(f_p/f_2)$, for a turbulent flow behind a grid ($R_\lambda = 5.3 \times 10^2$) and a turbulent flow from a jet ($R_\lambda = 8.0 \times 10^2$) [7]. The measured function f_p/f_2 is significantly different from one. At the smallest scales, $r \lesssim 30\eta$, f_p/f_2 increases with increasing p. Because the function f_p is convex, the result of Fig. 1 demonstrates that for $r \lesssim 30\eta$ f_p becomes flatter with increasing p. This behavior is consistent with the multiscaling hypothesis [3,8,9]. Because the p dependence of f_p/f_2 is weak, the result of Fig. 1 also indicates that for the even moments, $p \leq 8$, extended self-similarity is a good approximation. It is correct to within $|\log_{10}(f_p/f_2)| \lesssim 0.08$.

For large enough Reynolds numbers a flow approaches the situation of homogeneity and isotropy to which Kolmogorov's scaling laws apply. For the flows with the largest R_λ that exhibit a clear scaling behavior, we found that all normalizations produced the same $\zeta(p)$. Normalization $I_p(r)$ is clearly superior in the case that $G_p(r)$ does not exhibit obvious scaling. However, at small R_λ the questions about homogeneity and isotropy become urgent. Probably for reasons of statistics, Benzi *et al.* [4] compute normalization I_p with $\langle|\Delta u|^3\rangle$. It is not clear that $\langle|\Delta u|^3\rangle \sim r$. Indeed, we have found significant differences with the proper normalization that does *not* involve absolute values.

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[7] Grid turbulence was generated in a wind tunnel using a grid with mesh size 0.1 m and 0.36 solidity. The fluctuating velocity was measured with a hot-wire anemometer (0.2 mm probe size) 3 m behind the grid and off the center line. The jet flow emanated with 30 m/s from a 12 cm diameter jet. Data were taken 2.6 m downstream. In both cases the 12 bits of velocity data were acquired with a 20 kHz sampling rate and using a four pole antialiasing filter at 10 kHz.

[8] G. Stolovitsky and K.R. Sreenivasan, Phys. Rev. E **48**, 33 (1993) also find deviations from extended self-similarity in boundary-layer turbulence. The deviations found are in the opposite direction (towards a more strongly curved f_p as p increases). We have experimentally verified their conclusion and have found that the different behavior of f_p is caused by the proximity of the start of the boundary layer and by the proximity of the wall.

[9] The results shown here are typical for the results obtained in different turbulent flows. These will be presented in a forthcoming publication. The jet data have the smallest Kolmogorov length η; the first point at $r/\eta = 12$ in Fig. 1 is, therefore, at the edge of our instrumental time resolution. By also showing the grid turbulence data whose Kolmogorov frequency ($f_k = U/\eta$) is a factor of 2 smaller we demonstrate that the limitation on time resolution does not significantly influence our results. We have also systematically varied f_k in the jet experiment which produced consistent results for the residues. We have not been able to observe a dependence of the shape of $f_p(r)$ on R_λ. The multiscaling hypothesis predicts a very weak dependence on the log of R_λ [3].